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Technical Notes

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Velocity Spectrum Model for Turbulence Ingestion Noise from Computational-Fluid-Dynamics Calculations

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Nomenclature

c	=	Kolmogorov energy spectrum empirical constant
$E(\kappa)$	=	energy spectrum
$F_{11}(\kappa_1)$	=	longitudinal velocity spectrum
$F_{22}(\kappa_1)$	=	transverse velocity spectrum
$f(r)$	=	longitudinal velocity correlation function
$g(r)$	=	transverse velocity correlation function
k	=	turbulent kinetic energy
L_{11}	=	longitudinal integral length scale
$R_{ij}(\boldsymbol{r})$	=	velocity spatial correlation tensor
U_i	=	mean velocity vector
u	=	rms velocity
α	=	von Kármán energy spectrum constant
$\Gamma(x)$	=	Euler gamma function
δ	=	boundary-layer thickness
δ_{ij}	=	Kronecker delta function
ε	=	turbulent dissipation rate
κ	=	wave number
κ_e	=	wave number of energy-containing eddies
$\Phi_{ij}(\boldsymbol{\kappa})$	=	velocity spectrum tensor

I. Introduction

TURBULENCE ingested into a turbomachinery rotor or propeller interacts with the blades to generate unsteady forces and noise. Most analytical models of rotor or propeller turbulence-induced unsteady forces[1–4] assume that both the mean inflow velocity and the correlation (or spectrum) of the turbulent velocity fluctuations are known. The models focus on the prediction of the unsteady forces and noise in terms of the inflow velocity statistics, rotor geometry, and Sears gust response function.[5]

In many cases of practical importance, however, such as the design of a new turbomachine, the inflow to the rotor is not known and must be predicted in order to estimate the turbulence-induced unsteady forces and noise. Computational fluid dynamics (CFD), based on the Reynolds-averaged Navier–Stokes equations and a two-equation turbulence model, is often used to predict the mean velocity and pressure fields in turbomachinery. The same CFD solution can also be used to provide parameters for a turbulence velocity spectrum model. In brief, the turbulent kinetic energy k and dissipation rate ε are used in a model of the turbulence energy spectrum, which is then used to obtain the velocity spectrum and correlation tensors by assuming locally homogeneous and isotropic turbulence. An interesting result of this approach is that it provides an explicit relationship between the integral length scale and k–ε. Following the development of the model, its accuracy will be examined for the case of a flat-plate turbulent boundary layer, which could, for example, represent the inflow to a propeller operating in a hull boundary layer.

II. Turbulence Energy Spectrum Model

One of the most important results of classical turbulence theory[6,7] is that the functional form of the energy spectrum $E(\kappa)$ is reasonably independent of the class of flow (i.e., boundary layer, wake, jet, channel, etc.). Basically, the energy spectrum consists of three wave-number ranges: the large-scale range, which contains eddies whose sizes are on the order of the mean flow features; the inertial subrange, in which the energy is cascaded to smaller and smaller scales through the inviscid stretching and distortion of the eddies; and the dissipation range, in which viscous effects dominate. Kolmogorov[8] showed that the inertial subrange can depend only on ε and the wave number κ because all of the energy is supplied by the smaller wave numbers of the large-scale range and dissipated by the larger wave numbers of the dissipation range. Based on this hypothesis, the functional form of $E(\kappa)$ in the inertial subrange becomes

$$E(\kappa) = c\varepsilon^a \kappa^b \tag{1}$$

Using dimensional analysis, it is found that the exponents must be $a = \frac{2}{3}$ and $b = -\frac{5}{3}$. Furthermore, experiments have shown (e.g., Saddoughi and Veeravalli[9]) that the constant $c \simeq 1.5$.

A model for $E(\kappa)$ based on a $\kappa^{-5/3}$ high wave-number asymptote and a κ^4 dependence for low wave numbers was proposed by von Kármán[10]:

$$E(\kappa) = \alpha(k/\kappa_e)(\kappa/\kappa_e)^4\left[1 + (\kappa/\kappa_e)^2\right]^{-17/6} \tag{2}$$

The constant α can be determined because the integration of $E(\kappa)$ over all κ equals the kinetic energy, resulting in

$$\alpha = \frac{110}{27\sqrt{\pi}}\frac{\Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{1}{3}\right)} \simeq 0.97 \tag{3}$$

The parameter κ_e, which represents the wave number associated with the energy-containing eddies, can be expressed in terms of k and ε by equating the high wave-number asymptote of Eq. (2) with Eq. (1), yielding

$$\kappa_e = (c/\alpha)^{\frac{3}{2}}\left(\varepsilon/k^{\frac{3}{2}}\right) \simeq 1.9\left(\varepsilon/k^{\frac{3}{2}}\right) \tag{4}$$

Therefore, the von Kármán model can be used to predict $E(\kappa)$ given the k–ε output from a CFD solution.

III. Velocity Spectrum and Correlation Function

If the turbulence is assumed to be isotropic, all of the components of the velocity spectrum tensor Φ_{ij} can be obtained from the energy spectrum[7]

$$\Phi_{ij}(\boldsymbol{\kappa}) = \frac{E(\kappa)}{4\pi\kappa^2}\left(\delta_{ij} - \frac{\kappa_i\kappa_j}{\kappa^2}\right) \tag{5}$$

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The one-dimensional longitudinal spectrum $F_{11}(\kappa_1)$ and transverse spectrum $F_{22}(\kappa_1)$ can then be obtained by integration of the spectrum tensor over the κ_2–κ_3 plane. For the von Kármán energy spectrum model the results are[7]

$$F_{11}(\kappa_1) = \frac{9\alpha}{55}\frac{k}{\kappa_e}\left[1 + \left(\frac{\kappa_1}{\kappa_e}\right)^2\right]^{-\frac{5}{6}} \tag{6}$$

$$F_{22}(\kappa_1) = \frac{1}{2}\left(\frac{\kappa_e^2 + \frac{8}{3}\kappa_1^2}{\kappa_e^2 + \kappa_1^2}\right)F_{11}(\kappa_1) \tag{7}$$

The normalized inverse Fourier transforms of F_{11} and F_{22} yield the longitudinal and transverse correlation functions $f(r)$ and $g(r)$. These functions are normalized by the rms velocity u, which for isotropic turbulence is given by

$$u = \sqrt{(2/3)k} \tag{8}$$

because k is one-half the sum of the three components of the mean square velocity. The entire spatial correlation tensor R_{ij} can then be expressed in terms of u, $f(r)$, and $g(r)$:

$$R_{ij}(\boldsymbol{r}) = u^2\left[\left(r_i r_j / r^2\right) f(r) + \left(\delta_{ij} - r_i r_j / r^2\right) g(r)\right] \tag{9}$$

In addition to the rms velocity, the most common velocity statistic used in turbulence ingestion models is the longitudinal integral length scale, which is defined as

$$L_{11} = \int_0^\infty f(r)\,\mathrm{d}r \tag{10}$$

Because of the Fourier transform relationship between $f(r)$ and $F_{11}(\kappa_1)$, it follows that

$$F_{11}(0) = u^2 L_{11} / \pi \tag{11}$$

The integral length scale can be related to k–ε by using this result along with Eq. (6):

$$L_{11} = \frac{27\pi}{110}\frac{\alpha}{\kappa_e} = \frac{27\pi}{110}\left(\frac{\alpha^{\frac{5}{2}}}{c^{\frac{3}{2}}}\right)\frac{k^{\frac{3}{2}}}{\varepsilon} \simeq 0.39\frac{k^{\frac{3}{2}}}{\varepsilon} \tag{12}$$

IV. Example Application

To test the accuracy of this methodology for a case of practical interest, the velocity spectrum in a turbulent boundary layer was calculated. In a boundary layer the spectrum model assumption of isotropic turbulence is clearly violated. However, it will be shown that the model predictions are accurate for the inertial subrange and that the errors in the low wave-number spectrum are bounded by the anisotropy of the rms velocity and integral length scale.

The input to the model was generated using the commercial CFD code CFX-TASCflow. A model of a large two-dimensional channel was used in order to simulate a zero pressure gradient, with the channel half-height equal to approximately 100 times the boundary-layer thickness at the exit. The k–ε turbulence model was used along with wall functions for the wall boundary condition. A streamwise station was extracted from the solution to compare to the experimental measurements of Klebanoff.[11] The boundary-layer Reynolds number, based on momentum thickness, was about 7800 for both cases.

The longitudinal and transverse wave-number spectra were predicted for the height $y/\delta = 0.58$, where the model parameters were predicted to be $u = 0.035U_\infty$ and $L_{11} = 0.24\delta$ based on the CFD output values of k and ε. Figure 1 shows that the predicted spectra are within 1 dB of Klebanoff's measured spectra for the inertial subrange, implying that the isotropic model is valid in this range and that the model parameters are predicted accurately. In addition, the faster roll-off of the measured spectra at higher wave numbers is easily explained, because the spectrum model does not include dissipation effects. A model of the dissipation range could possibly be added[6]; however, in most turbulence ingestion applications the Reynolds number is much higher than it is in this example, and so the dissipation range does not usually occur until very high wave numbers. Because trailing-edge noise tends to dominate the corresponding high-frequency range, it is not usually necessary to model the dissipation range for turbulence ingestion applications.

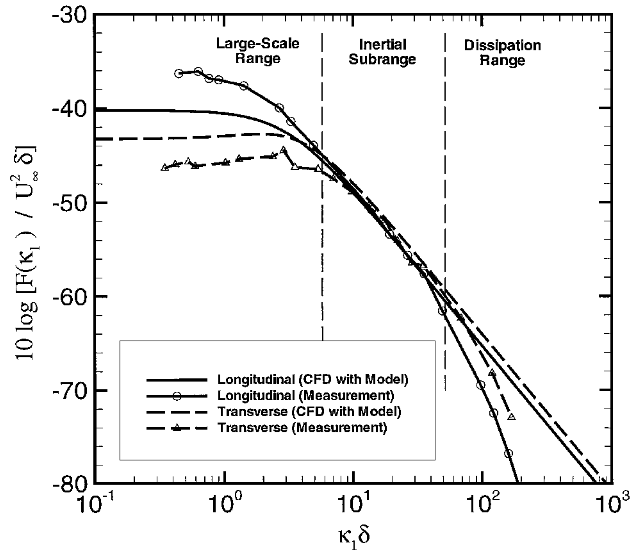

Fig. 1 Predicted vs measured longitudinal and transverse wave-number spectra.

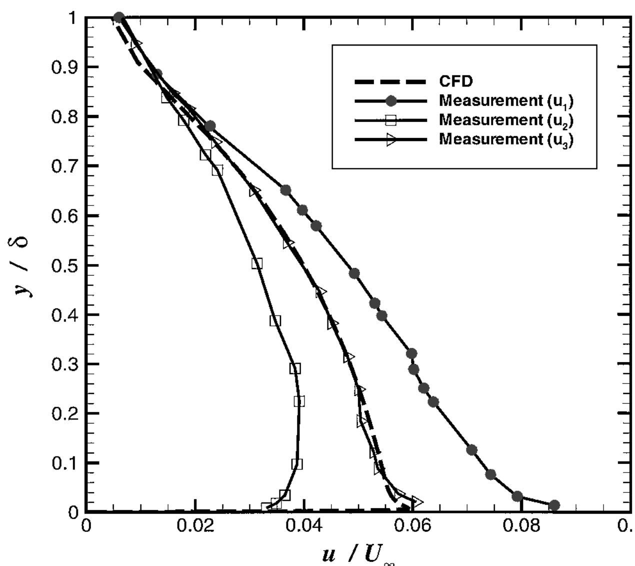

Fig. 2 Predicted vs measured fluctuating velocity.

For low wave numbers, however, the effect of anisotropy is apparent, and the model underpredicts the longitudinal spectrum by about 4 dB and overpredicts the transverse spectrum by about 3 dB. This discrepancy is not surprising given the profiles of the three components of the rms velocity, as shown in Fig. 2. The CFD result is a good representation of the average rms velocity and is very close to the measured spanwise component u_3. However, the streamwise component u_1 used in the longitudinal spectrum is higher than the average, and the wall-normal component u_2 used in the transverse spectrum is lower than the average. At $y/\delta = 0.58$ the measured values are $u_1 = 0.042U_\infty$ and $u_2 = 0.028U_\infty$ (Ref. 11). Using these results in Eq. (11), with the appropriate change of subscript for the transverse component, yields integral length scales of $L_{11} = 0.45\delta$ and $L_{22} = 0.095\delta$. Gavin[4] reported an identical value of L_{11} in a zero-pressure-gradient turbulent boundary layer whose momentum thickness Reynolds number was 4.1×10^3, but did not report any value of L_{22}. Using these anisotropic values of the rms velocity and length scale in Eq. (11) results in the measured 10 dB spread between the low-wave-number longitudinal and transverse spectra. However, in an isotropic model $u_1 = u_2$ and $L_{11} = 2L_{22}$, resulting in only a 3-dB spread. Therefore, the best possible fit for an isotropic model is a $(10 - 3)/2 = 3.5$ dB underprediction of the longitudinal spectrum and overprediction of the transverse spectrum. This is very close to what is obtained by the CFD-based prediction.

Although the predicted low-wave-number spectrum using the isotropic model has errors of 3–4 dB in a boundary layer, the effect of anisotropy on turbulence ingestion noise calculations is likely to be less significant. This is because unsteady lift is produced by the fluctuating velocity component normal to the blade, which, because of the blade stagger angle, rake, and skew, is generally a combination of u_1, u_2, and u_3 (Ref. 4). Therefore, an isotropic model based on an average of the three velocity components is likely to provide a reasonable approximation of the blade-normal velocity spectrum for rotor turbulence ingestion noise predictions.

V. Conclusions

A model that can be used to predict the turbulence velocity spectrum and correlation function given the k–ε output of a CFD solution has been presented. The model is most accurate in the inertial subrange because of the assumption of isotropy, as demonstrated for a turbulent boundary layer. Because the model is based on the turbulence energy spectrum, it is reasonably independent of the class of flow. For use in predicting turbulence ingestion noise in turbomachinery, the model should provide a reasonable approximation for the entire wave-number range regardless of whether the turbulence is caused by a hull boundary layer, a wake from an upstream obstruction, or a jet from an upstream impeller.

References

[1]Sevik, M., "Sound Radiation from a Subsonic Rotor Subjected to Turbulence," *Fluid Mechanics, Acoustics, and Design of Turbomachinery, Part II*, NASA SP-304, 1974, pp. 493–512.

[2]Thompson, D. E., "Propeller Time Dependent Forces Due to Nonuniform Flow," Ph.D. Dissertation, Dept. of Aerospace Engineering, Pennsylvania State Univ., University Park, PA, May 1976.

[3]Wojno, J. P., Mueller, T. J., and Blake, W. K., "Turbulence Ingestion Noise, Part 2: Rotor Aeroacoustic Response to Grid-Generated Turbulence," *AIAA Journal*, Vol. 40, No. 1, 2002, pp. 26–32.

[4]Gavin, J. R., "Unsteady Forces and Sound Caused by Boundary Layer Turbulence Entering a Turbomachinery Rotor," Ph.D. Dissertation, Graduate Program in Acoustics, Pennsylvania State Univ., University Park, PA, Aug. 2002.

[5]Sears, W. R., "Some Aspects of Non-Stationary Airfoil Theory and Its Practical Application," *Journal of the Aeronautical Sciences*, Vol. 8, 1941, pp. 104–108.

[6]Tennekes, H., and Lumley, J. L., *A First Course in Turbulence*, MIT Press, Cambridge, MA, 1972, Chap. 8.

[7]Hinze, J. O., *Turbulence*, McGraw–Hill, New York, 1975, Chap. 3.

[8]Kolmogorov, A. N., "The Local Structure of Turbulence in Incompressible Viscous Fluid for Very Large Reynolds Numbers," *Doklady Academii Nauk SSR*, Vol. 30, 1941, pp. 301–305; translated by V. Levin and reprinted in *Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences*, Vol. 434, 1991, pp. 9–13.

[9]Saddoughi, S. G., and Veeravalli, S. V., "Local Isotropy in Turbulent Boundary Layers at High Reynolds Number," *Journal of Fluid Mechanics*, Vol. 268, 1994, pp. 333–372.

[10]von Kármán, T., "Progress in the Statistical Theory of Turbulence," *Proceedings of the National Academy of Science (USA)*, Vol. 34, 1948, pp. 530–539.

[11]Klebanoff, P. S., "Characteristics of Turbulence in a Boundary Layer with Zero Pressure Gradient," *National Advisory Committee for Aeronautics*, Rept. 1247, U.S. Government Printing Office, Washington, DC, 1956.

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Numerical Simulation of Shock Oscillations over Airfoil Using a Wall Law Approach

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I. Introduction

SHOCK-INDUCED oscillations (SIO) are a pure aerodynamic problem that can occur over rigid airfoils as a result of the development of instabilities caused by the boundary-layer separation and the shock-wave interaction. This problem is extremely important because it can lead to the buffeting phenomenon through the mechanical response of the wing structure. A detailed description of the physical features of SIO is given by Lee.[1] Computations have been essentielly done over thick airfoils to investigate the SIO problem.[2–4]

Numerically, the local time-step technique, which is efficient to accelerate the convergence towards the steady state, cannot be applied for unsteady computations that imperatively need a global time step. This constraint drastically reduces the method efficiency because of the Courant–Friedrichs–Lewy stability criterion. To overcome this difficulty, the dual time-stepping approach has been proposed by Jameson. It has also been used recently by Furlano et al.[5] and Renaud et al.[6] for SIO computations.

In the present study a wall law approach is used to relax the mesh refinement near the wall and therefore to increase the value of the global time step. Then, the computational efficiency of the explicit method is restored. Providing large CPU cost savings, the wall law approach has also proved to be attractive for the quality of its results in computing separated flows and for the robustness improvment it brings.[7]

Moreover, the eddy viscosity models based on the linear Boussinesq relation are known to be unable to capture the boundary-layer separation and unable to take into consideration the nonequilibrium effects. A consequence of these weakness observed for unsteady computations is the overproduction of eddy viscosity, which limits the development of natural unsteadiness and modifies the flow topology. In this Note, we show that the turbulence model behavior can be remarkably improved by limiting the eddy viscosity with the shear-stress-transport (SST) correction associated with a wall law approach.

II. Numerical Methods

A code solving the uncoupled Reynolds-averaged Navier–Stokes/turbulent systems for multidomain structured meshes is used for the present study. This code is based on a cell-centered finite volume discretization. Fluxes are computed with the Jameson scheme, and time integration is realized through a four-stage Runge–Kutta algorithm. More details about the solver can be found in Ref. 8.

Various two-equation turbulence models are used in the present study: the Menter[9] SST k–ω model and also the high-Reynolds-number version of the Jones–Launder[10] k–ε model, with and without the Menter SST correction.

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